Neutrino Oscillation and Charged Lepton-Flavor Violation in the Supersymmetric Standard Models

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The neutrino experiment results suggest that the neutrinos have finite masses and the lepton-flavor symmetries are violating in nature. In the supersymmetric models, the charged lepton-flavor violating processes, such as $\mu^+ \to e^+\gamma$ and $\tau^\pm \to \mu^\pm\gamma$, may have the branching ratios accessible to the future experiments, depending on origins of the neutrino masses and the SUSY breaking. In this paper we discuss the branching ratios in the supergravity scenario using the current solar and atmospheric neutrino experimental data.

1 Introduction

Result of the atmospheric neutrino experiment by the superKamiokande detector indicates that the neutrinos have finite masses and the lepton-flavor symmetry of muon is violating in nature [1]. This is the first signature of the physics beyond the standard model (SM), and this discovery will be confirmed by further experiments, such as the long base-line experiments. Also, the solar neutrino experimental data suggest that the lepton-flavor symmetry of electron is violating [2].

Processes, such as $\mu^+ \to e^+\gamma$ and $\tau^\pm \to \mu^\pm\gamma$, are also lepton-flavor violating (LFV) processes. Unfortunately, the event rates are too small to be observed in near future experiments even if the neutrino masses are introduced into the standard model. The event rates are suppressed by the fourth order of the ratio of the tiny neutrino mass to the W boson mass due to the GIM suppression. However, if the standard model is supersymmetrized, the processes may be accessible in near future experiment and we may study the origin of the neutrino masses.

The supersymmetric standard model (SUSY SM) is a solution of the naturalness problem, and is one of the most promising model beyond the standard model. In this model, introduction of the SUSY breaking terms allows the lepton-flavor symmetries to be violating in the slepton masses [3]. Then, the orders of magnitude of the event rates for the LFV processes depend on the origin of the SUSY breaking in the SUSY SM and physics beyond the SUSY

SM. One of the successful candidates for the origin of the SUSY breaking is the minimal supergravity, where the SUSY breaking scalar masses are generated universally in the flavor space at the tree level. In this scenario, the source of the LFV processes comes from the LFV radiative correction to the SUSY breaking masses for the sleptons by the LFV interaction in physics beyond the SUSY SM[4]. Then, we have a chance to study the origin of the neutrino masses through the LFV processes in the supersymmetric models.

The see-saw mechanism[5] is the simplest model to generate the tiny neutrino masses. In this mechanism the Yukawa interactions are lepton-flavor violating due to introduction of the right-handed neutrinos, similar to the quark sector. Then, in the minimal supergravity scenario, if the lepton-flavor violation in the Yukawa coupling constants is strong enough, the radiative correction generates sizable LFV masses for the sleptons[67]. Moreover, the large mixing angles observed on the solar and atmospheric neutrino observations may enhance the event rates for the LFV processes[8910].

In this article, we study the charged lepton-flavor violating processes, $\mu^+ \to e^+\gamma$ and $\tau^\pm \to \mu^\pm\gamma$, using the current neutrino experimental data, in the SUSY SM with the right-handed neutrinos.[a] We assume the minimal supergravity scenario. The large mixing angle in the atmospheric neutrino result may enhance $\tau^\pm \to \mu^\pm\gamma$, and the large mixing angles in the MSW and the vacuum oscillation solutions may lead to a large event rate of $\mu^+ \to e^+\gamma$.

This article is organized as follows. In the next section we review the radiative generation of the LFV masses for slepton in the SUSY SM with the right-handed neutrinos. In Section 3 we show the branching rate for $\tau^\pm \to \mu^\pm\gamma$, using the atmospheric neutrino result. In Section 4 discuss the branching rate for $\mu^+ \to e^+\gamma$, using the solar neutrino result. The other processes are also discussed. Section 5 is devoted to Conclusion.

2 The SUSY SM with the right-handed neutrinos

We review the radiative generation of the LFV masses for sleptons in the SUSY SM with the right-handed neutrinos. We adopt the minimal supergravity scenario as the origin of the SUSY breaking in the SUSY SM. The superpotential of the lepton sector in the SUSY SM with right-handed neutrinos is given as

$$W = f_{\nu_{ij}} H_2 \overline{N}_i L_j + f_{e_{ij}} H_1 \overline{E}_i L_j + \frac{1}{2} M_{\nu_i \nu_j} \overline{N}_i \overline{N}_j, \tag{1}$$

where L is a chiral superfield for the left-handed lepton, and $\overline{N}$ and $\overline{E}$ are for the right-handed neutrino and the charged lepton. H_1 and H_2 are for the

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[a] After sleptons are discovered in the future large colliders, the slepton oscillation will be a powerful tool to study the lepton-flavor violation[11].

Higgs doublets in the SUSY SM. Here, i and j are generation indices. After redefinition of the fields, the Yukawa coupling constants and the Majorana masses can be taken as

$$
\begin{aligned}
f_{\nu_{ij}} &= f_{\nu_i} V_{Dij}, \\
f_{e_{ij}} &= f_{e_i} \delta_{ij}, \\
M_{\nu_i \nu_j} &= U_{ik}^* M_{\nu_k} U_{kj}^\dagger,
\end{aligned}
\tag{2}
$$

where V_D and U are unitary matrices. In this model the mass matrix for the left-handed neutrinos (m_ν) becomes

$$
(m_\nu)_{ij} = V_{Dik}^\top (\overline{m}_\nu)_{kl} V_{Dlj},
\tag{3}
$$

where

$$
\begin{aligned}
(\overline{m}_\nu)_{ij} &= m_{\nu_i D} \left[M^{-1} \right]_{ij} m_{\nu_j D} \\
&\equiv V_{Mik}^\top m_{\nu_k} V_{Mkj}.
\end{aligned}
\tag{4}
$$

Here, $m_{\nu_i D} = f_{\nu_i} v \sin\beta/\sqrt{2}$ and V_M is a unitary matrix.[b] The observed mixing angles on the atmospheric and solar neutrino experiments are $(V_M V_D)_{\tau\mu}$ and $(V_M V_D)_{\mu e}$, respectively, if they come from the oscillations of $\nu_\mu - \nu_\tau$ and $\nu_e - \nu_\mu$.

The SUSY breaking terms for the lepton sector in the SUSY SM with the right-handed neutrinos are in general given as

$$
\begin{aligned}
-\mathcal{L}_{\text{SUSY breaking}} = \quad & (m_{\tilde{L}}^2)_{ij} \tilde{l}_{Li}^\dagger \tilde{l}_{Lj} + (m_{\tilde{e}}^2)_{ij} \tilde{e}_{Ri}^* \tilde{e}_{Rj} + (m_{\tilde{\nu}}^2)_{ij} \tilde{\nu}_{Ri}^* \tilde{\nu}_{Rj} \\
& + (A_\nu^{ij} h_2 \tilde{\nu}_{Ri}^* \tilde{l}_{Lj} + A_e^{ij} h_1 \tilde{e}_{Ri}^* \tilde{l}_{Lj} + \frac{1}{2} B_\nu^{ij} \tilde{\nu}_{Ri}^* \tilde{\nu}_{Rj}^* + h.c.),
\end{aligned}
\tag{5}
$$

where $\tilde{l}_L$, $\tilde{e}_R$, and $\tilde{\nu}_R$ represent the left-handed slepton, and the right-handed charged slepton, and the right-handed neutrino. Also, h_1 and h_2 are the doublet Higgs bosons. In the minimal supergravity scenario the SUSY breaking masses for sleptons, squarks, and the Higgs bosons are universal at the gravitational scale ($M_{\text{grav}} \sim 10^{18}$ GeV), and the SUSY breaking parameters associated with the supersymmetric Yukawa couplings or masses (A or B parameters) are proportional to the Yukawa coupling constants or masses. Then, the SUSY breaking parameters in Eq. (5) are given as

$$
\begin{aligned}
(m_{\tilde{L}}^2)_{ij} = (m_{\tilde{e}}^2)_{ij} = (m_{\tilde{\nu}}^2)_{ij} &= \delta_{ij} m_0^2, \\
A_\nu^{ij} = f_{\nu_{ij}} a_0, \quad A_e^{ij} &= f_{e_{ij}} a_0, \\
B_\nu^{ij} &= M_{\nu_i \nu_j} b_0,
\end{aligned}
\tag{6}
$$

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[b] $\langle h_1 \rangle = (v \cos\beta/\sqrt{2}, 0)^\top$ and $\langle h_2 \rangle = (0, v \sin\beta/\sqrt{2})^\top$ with $v \simeq 246$ GeV.

at the tree level.

In order to know the values of the SUSY breaking parameters at the low energy, we have to include the radiative corrections to them. While we evaluate them by solving the RGE's, we discuss only the qualitative behavior of the solution using the logarithmic approximation here. The SUSY breaking masses of squarks, sleptons, and the Higgs bosons at the low energy become heavier by gauge interactions at one-loop level, and the corrections are flavor-independent. On the other hand, Yukawa interactions reduce the diagonal SUSY breaking mass squareds and the radiative corrections are flavor-dependent. Then, if the Yukawa coupling is lepton-flavor violating, the radiative correction to the SUSY breaking parameters is also lepton-flavor violating. The LFV off-diagonal components for $(m_{\tilde{L}}^2)$, $(m_{\tilde{e}}^2)$, and A_e in the SUSY SM with the right-handed neutrinos are given at the low energy as

$$(m_{\tilde{L}}^2)_{ij} \simeq -\frac{1}{8\pi^2}(3m_0^2 + a_0^2)V_{Dki}^*V_{Dlj}f_{\nu_k}f_{\nu_l}U_{km}^*U_{lm}\log\frac{M_{\mathrm{grav}}}{M_{\nu_m}},$$

$$(m_{\tilde{e}}^2)_{ij} \simeq 0,$$

$$A_e^{ij} \simeq -\frac{3}{8\pi^2}a_0f_{e_i}V_{Dki}^*V_{Dlj}f_{\nu_k}f_{\nu_l}U_{km}^*U_{lm}\log\frac{M_{\mathrm{grav}}}{M_{\nu_m}}, \tag{7}$$

where $i \neq j$. In these equations, the off-diagonal components of $(m_{\tilde{L}}^2)$ and A_e are generated radiatively while those of $(m_{\tilde{e}}^2)$ are not. This is because the right-handed leptons have only one kind of the Yukawa interaction f_e and we can always take a basis where f_e is diagonal. This is a characteristic of the SUSY SM with the right-handed neutrinos. [c]

The magnitudes of the off-diagonal components of $(m_{\tilde{L}}^2)$ and A_e are sensitive to f_{ν_i} and V_D, while not to U. This is because the off-diagonal components of U are small when the hierarchy among the right-handed neutrino masses is large, and then we will take $U = \mathbf{1}$ in the following discussion for simplicity. In the following sections we will evaluate the values of f_{ν_i} and V_D from the neutrino oscillation data.

3 The atmospheric neutrino result and $\tau^\pm \to \mu^\pm\gamma$

In this section we discuss the branching ratio of $\tau^\pm \to \mu^\pm\gamma$ using the atmospheric neutrino result. From the zenith-angle dependence of ν_e and ν_μ fluxes

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[c] In the minimal SU(5) SUSY GUT, the right-handed sleptons receive the LFV masses through the Yukawa interaction of colored Higgs, but not the left-handed ones [12] [13]. In the SO(10) SUSY GUT and the non-minimal SU(5) SUSY GUT both sleptons may have the LFV masses [14].
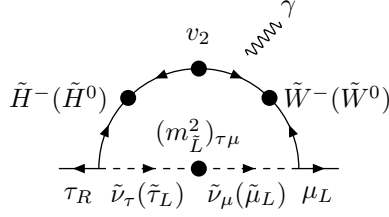
Figure 1: The Feynman diagram which gives a dominant contribution to $\tau^+ \to \mu^+ \gamma$ when $\tan\beta \gtrsim 1$ and the off-diagonal elements of the right-handed slepton mass matrix are negligible, as in the MSSM with the right-handed neutrinos. $\tilde{\tau}_L$ and $\tilde{\mu}_L$ are the left-handed stau and smuon, respectively, and $\tilde{\nu}_\tau$ and $\tilde{\nu}_\mu$ the tau sneutrino and the mu sneutrino. $\tilde{H}$ is Higgsino, $\tilde{W}$ wino. The arrows represent the chiralities.
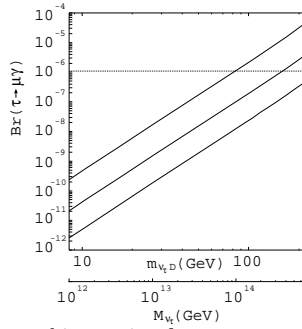

Figure 2: Dependence of the branching ratio of $\tau \to \mu\gamma$ on the Dirac neutrino mass for the tau neutrino $m_{\nu_\tau D}$ (the right-handed tau neutrino mass M_{ν_τ}). The dotted line is the current experimental bound. Here, $m_{\nu_\tau} = 0.07$eV, $\sin 2\theta_D = 1$. Also, we take $m_{\tilde{e}_L} = 170$GeV and the wino mass 130GeV. The other gaugino masses are determined by the GUT relation for the gaugino masses for simplicity. Also, we impose the radiative breaking condition of the $SU(2)_L \times U(1)_Y$ gauge symmetries with $\tan\beta = 3, 10, 30$ and the Higgsino mass parameter positive. Here also the larger $\tan\beta$ corresponds to the upper line.

5

measured by the superKamiokande it is natural that the atmospheric neutrino anomaly comes from the neutrino oscillation between ν_μ and ν_τ, and the neutrino mass-squared difference and mixing angle are expected as

$$\Delta m^2_{\nu_\mu \nu_\tau} \simeq 10^{-(2-3)} \text{eV}^2,$$
$$\sin^2 2\theta_{\nu_\mu \nu_\tau} \gtrsim 0.8. \tag{8}$$

Assuming that the neutrino masses is hierarchical as $m_{\nu_\tau} \gg m_{\nu_\mu} \gg m_{\nu_e}$, the tau neutrino mass is given as

$$m_{\nu_\tau} \simeq (3 \times 10^{-2} - 1 \times 10^{-1}) \text{eV}, \tag{9}$$

and if the tau neutrino Yukawa coupling constant f_{ν_τ} is as large as that of the top quark, the right-handed tau neutrino M_{ν_τ} is about 10^{14-15}GeV.

In order to evaluate the event rate for $\tau^\pm \to \mu^\pm \gamma$, we have to know the value of $V_{D\tau\mu}$, which is not necessary the same as the $\sin\theta_{\nu_\mu \nu_\tau}$. However, it is expected that it is also of the order of one as explained bellow.

Let us consider only the tau and the mu neutrino masses for simplicity. In this case we parameterize two unitary matrices V_D and V_M as

$$V_D = \begin{pmatrix} \cos\theta_D & \sin\theta_D \\ -\sin\theta_D & \cos\theta_D \end{pmatrix}, \quad V_M = \begin{pmatrix} \cos\theta_M & \sin\theta_M \\ -\sin\theta_M & \cos\theta_M \end{pmatrix}. \tag{10}$$

The observed large angle $\theta_{\nu_\mu \nu_\tau}$ is a sum of θ_D and θ_M. However, in order to derive $\theta_M \sim \pi/4$ we need a fine-tune among the independent Yukawa coupling constants and the mass parameters. The neutrino mass matrix $(\overline{m}_\nu)$ for the second and the third generations (Eq. (3)) is given as

$$(\overline{m}_\nu) \propto \begin{pmatrix} \dfrac{m^2_{\nu_\mu D}}{M_{\nu_\mu \nu_\mu}} & -\dfrac{m_{\nu_\mu D} m_{\nu_\tau D}}{M_{\nu_\mu \nu_\tau}} \dfrac{M^2_{\nu_\mu \nu_\tau}}{M_{\nu_\mu \nu_\mu} M_{\nu_\tau \nu_\tau}} \\ -\dfrac{m_{\nu_\mu D} m_{\nu_\tau D}}{M_{\nu_\mu \nu_\tau}} \dfrac{M^2_{\nu_\mu \nu_\tau}}{M_{\nu_\mu \nu_\mu} M_{\nu_\tau \nu_\tau}} & \dfrac{m^2_{\nu_\tau D}}{M_{\nu_\tau \nu_\tau}} \end{pmatrix}. \tag{11}$$

If the following relations are valid,

$$\frac{m^2_{\nu_\tau D}}{M_{\nu_\tau \nu_\tau}} \simeq \frac{m^2_{\nu_\mu D}}{M_{\nu_\mu \nu_\mu}} \simeq \frac{m_{\nu_\mu D} m_{\nu_\tau D}}{M_{\nu_\mu \nu_\tau}}, \tag{12}$$

$m_{\nu_\tau} \gg m_{\nu_\mu}$ and $\theta_M \simeq \pi/4$ can be derived. However, the relation among the independent coupling constants and masses is not natural without some mechanism or symmetry. Also, if $m_{\nu_\tau D} \gg m_{\nu_\mu D}$ similar to the quark sector, the mixing angle θ_M tends to be suppressed as

$$\tan 2\theta_M \simeq 2 \left(\frac{m_{\nu_\mu D}}{m_{\nu_\tau D}} \right) \left(\frac{M_{\nu_\mu \nu_\tau}}{M_{\nu_\mu \nu_\mu}} \right). \tag{13}$$

6

Therefore, in the following discussion we assume that the large mixing angle between ν_τ and ν_μ comes from θ_D and that V_M is a unit matrix.

Large $V_{D\tau\mu}$ leads to the non-vanishing $(m_{\tilde{L}}^2)_{\tau\mu}$ and $A_e^{\tau\mu}$, which result in a finite $\tau^\pm \to \mu^\pm\gamma$ event rate via diagrams involving them. They are given as

$$(m_{\tilde{L}}^2)_{\tau\mu} \simeq \frac{1}{16\pi^2}(3m_0^2 + a_0^2)\sin 2\theta_D f_{\nu_\tau}^2 \log\frac{M_{\mathrm{grav}}}{M_{\nu_\tau}},$$

$$A_e^{\tau\mu} \simeq \frac{3}{16\pi^2}a_0 \sin 2\theta_D f_\tau f_{\nu_\tau}^2 \log\frac{M_{\mathrm{grav}}}{M_{\nu_\tau}}. \tag{14}$$

As will be shown, if f_{ν_τ} is of the order of one, the branching ratio of $\tau \to \mu\gamma$ may reach the present experimental bound.

Let us evaluate the branching ratios of $\tau \to \mu\gamma$. The amplitude of the $e_i^+ \to e_j^+\gamma$ $(i > j)$ takes a form

$$T = e\epsilon^{\alpha*}(q)\bar{v}_i(p)i\sigma_{\alpha\beta}q^\beta(A_L^{(ij)}P_L + A_R^{(ij)}P_R)v_j(p-q), \tag{15}$$

where p and q are momenta of e_i and photon, and the event rate is given by

$$\Gamma(e_i \to e_j\gamma) = \frac{e^2}{16\pi}m_{e_i}^3(|A_L^{(ij)}|^2 + |A_R^{(ij)}|^2). \tag{16}$$

Here, we neglect the mass of e_j. The amplitude is not invariant for the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ symmetries and the chiral symmetries of leptons. Then the coefficients $A_L^{(ij)}$ and $A_R^{(ij)}$ are proportional to the charged lepton masses. Since the mismatch between the left-handed slepton and the charged lepton mass eigenstates is induced in the SUSY SM with the right-handed neutrinos, $A_L^{(ij)}$ is much larger than $A_R^{(ij)}$. Also, when $\tan\beta(\equiv v_2/v_1)$ is large, the contribution to $A_L^{(ij)}$ proportional to $f_{e_i}v_2(= -\sqrt{2}m_{e_i}\tan\beta)$ becomes dominant. Then, the dominant contribution to $\tau^\pm \to \mu^\pm\gamma$ is from the diagram of Fig. (1) for $\tan\beta \gtrsim 1$.

In Fig. (2) we show the branching ratio of $\tau^\pm \to \mu^\pm\gamma$ as a function of the Dirac neutrino mass for tau neutrino $m_{\nu_\tau D}$ (the right-handed tau neutrino mass M_{ν_τ}). Here, $m_{\nu_\tau} = 0.07\mathrm{eV}$, $\sin 2\theta_D = 1$. Also, we take $m_{\tilde{e}_L} = 170\mathrm{GeV}$ and the wino mass $130\mathrm{GeV}$. The other gaugino masses are determined by the GUT relation for the gaugino masses for simplicity. Also, we impose the radiative breaking condition of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetries with $\tan\beta = 3, 10, 30$ and the Higgsino mass parameter positive. The branching ratio is proportional to $m_{\nu_\tau D}^4$ $(M_{\nu_\tau}^2)$. The current experimental bound is $Br \leq 1.1 \times 10^{-6}$ [15], and some region is excluded by it. If 10^{-8} can be reached in the future experiments, such as B factories, we can probe $m_{\nu_\tau D} > 20(80)\mathrm{GeV}$ for $\tan\beta = 30(3)$. Then, if the Dirac tau neutrino mass is as large as the top quark mass, we may observe $\tau^\pm \to \mu^\pm\gamma$ there.
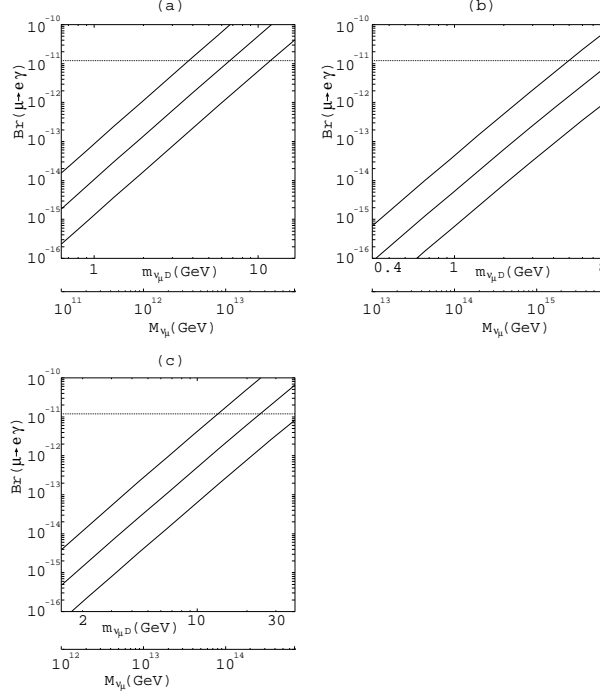
Figure 3: Dependence of the branching ratio of $\mu \to e\gamma$ on the Dirac neutrino mass for the mu neutrino $m_{\nu_\mu D}$ (the right-handed mu neutrino mass M_{ν_μ}). Here, a) is for the MSW solution with the large angle, b) is the vacuum oscillation solution, and c) is for the MSW solution with the small angle. m_{ν_μ} and V_D are given in text. The other parameters are the same as in Fig. (2). The dotted lines are the current experimental bound.

4   The solar neutrino result and $\mu^+ \to e^+\gamma$

In this section we discuss the relation between the solar neutrino result and $\mu^+ \to e^+\gamma$, assuming that the solar neutrino deficit comes from the $\nu_e - \nu_\mu$ oscillation. The relation is more complicated compared with that between the atmospheric neutrino result and $\tau^\pm \to \mu^\pm\gamma$.

There are three candidates for the solution of the solar neutrino deficit if it comes from neutrino oscillation. The MSW solution [16] due to the matter effect in the sun gives the natural explanation, and the observation favors

$$\Delta m^2_{\nu_e \nu_Y} \simeq 10^{-(4-5)}\text{eV}^2 \quad \text{or} \quad 10^{-7}\text{eV}^2,$$

8

$$\sin^2 2\theta_{\nu_e \nu_Y} \gtrsim 0.5, \qquad (17)$$

or

$$\Delta m^2_{\nu_e \nu_Y} \simeq 10^{-5} \text{eV}^2,$$
$$\sin^2 2\theta_{\nu_e \nu_Y} \simeq 10^{-(2-3)}. \qquad (18)$$

If the solar neutrino anomaly comes from so-called 'just so' solution [17], the neutrino oscillation in vacuum, the mass-squared difference and mixing angle are expected as [17]

$$\Delta m^2_{\nu_e \nu_Y} \simeq 10^{-(10-11)} \text{eV}^2,$$
$$\sin^2 2\theta_{\nu_e \nu_Y} \gtrsim 0.5. \qquad (19)$$

Assuming that the neutrino masses hierarchical as $m_{\nu_\tau} \gg m_{\nu_\mu} \gg m_{\nu_e}$, it is natural to consider $\nu_Y = \nu_\mu$. If one of the large angle solutions for the solar neutrino anomaly is true, the large mixing $\theta_{\nu_\mu \nu_e}$ may imply the LFV large mixing for sleptons between the first- and the second-generations. Similar to the atmospheric neutrino case, it is natural to consider that the large mixing angle between ν_μ and ν_e in the MSW solution or the 'just so' solution for the solar neutrino anomaly comes from V_D.

The amplitude for $\mu^+ \to e^+ \gamma$ is proportional to $(m^2_{\tilde{L}})_{\mu e}$, and it has two contributions in the SUSY SM with right-handed neutrinos as

$$(m^2_{\tilde{L}})_{\mu e} \simeq -\frac{1}{8\pi^2}(3m_0^2 + a_0^2) \times$$
$$\left(V^*_{D\tau\mu} V_{D\tau e} f^2_{\nu_\tau} \log \frac{M_{\text{grav}}}{M_{\nu_\tau}} + V^*_{D\mu\mu} V_{D\mu e} f^2_{\nu_\mu} \log \frac{M_{\text{grav}}}{M_{\nu_\mu}} \right). \qquad (20)$$

Here, we assume $f_{\nu_\tau} \gg f_{\nu_\mu} \gg f_{\nu_e}$, and the term proportional to $f^2_{\nu_e}$ is neglected. Unfortunately, we do not have information about $V_{D\tau e}$ and we can not evaluate the first term in Eq. (20). On the other hand, we can evaluate the second term if $V_{D\mu e}$ can be determined from the solar neutrino result. Then, in the following, we evaluate the event rate for $\mu^+ \to e^+ \gamma$ assuming $V_{D\tau e} = 0$. Notice that though this gives the conservative value for the event rate, there are also possibilities where the event rate is larger or smaller due to the finite $V_{D\tau e}$.

Let us evaluate $\mu^+ \to e^+ \gamma$. The forms of the amplitude and the event rate are the same as those of $\tau^\pm \to \mu^\pm \gamma$ (Eqs. (15,16)). As mentioned above, if the solar neutrino anomaly comes from the MSW effect or the vacuum oscillation

9

with the large angle, $V_{D\mu e}$ is expected to be large. This may lead to large $(m_{\tilde{L}}^2)_{\mu e}$. In Fig. (3-a), under the condition that

$$V_D = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{21}$$

we show the branching ratio of $\mu^+ \to e^+\gamma$ as a function of $m_{\nu_\mu D}$ (M_{ν_μ}). We take $m_{\nu_\mu} = 4.0 \times 10^{-3}$eV, which is consistent with the MSW solution. The other input parameters are taken to be the same as in Fig. (2). The branching ratio is promotional to $m_{\nu_\mu D}^4$ ($M_{\nu_\mu}^2$). For $\tan\beta = 30(3)$, the branching ratio reaches the experimental bound ($\mathrm{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [18]) when $m_{\nu_\mu D} \simeq 4(10)$GeV. A future experiment at PSI is expected to reach 10^{-14} [19]. This corresponds to $m_{\nu_\mu D} \simeq 0.5(2)$GeV. If we take $m_{\nu_\mu} = 1.0 \times 10^{-5}$eV expected by the 'just so' solution (Fig. (3-b)), the branching ratio becomes slightly smaller for a fixed $m_{\nu_\mu D}$ since the log factor in Eq. (20) is smaller.

If the solar neutrino anomaly comes from the MSW solution with the small mixing, we cannot distinguish whether the mixing comes from V_D or V_M even if using argument of naturalness. If it comes from V_D, the branching ratio is smaller by about $1/100$ compared with that in the MSW solution with the large mixing, as shown in Fig. (3-c). In Fig. (3-c) we assume that

$$V_D = \begin{pmatrix} 1 & 0.04 & 0.03 \\ -0.04 & 0.79 & 0.59 \\ 0 & -0.60 & 0.80 \end{pmatrix} \tag{22}$$

and $m_{\nu_\mu} = 2.2 \times 10^{-3}$eV. Other input parameters are the same as Fig. (2).

Finally we consider the $\mu^+ \to e^+e^-e^+$ process and the μ-e conversion in nuclei. For these processes the penguin type diagrams tend to dominate over the others, so the behavior of the decay rate is similar to that of $\mu^+ \to e^+\gamma$. For the $\mu^+ \to e^+e^-e^+$ process the following approximate relation holds,

$$\mathrm{Br}(\mu \to 3e) \simeq \frac{\alpha}{8\pi}\frac{8}{3}\left(\log\frac{m_\mu^2}{m_e^2} - \frac{11}{4}\right)\mathrm{Br}(\mu \to e\gamma) \tag{23}$$

$$\simeq 7 \times 10^{-3}\mathrm{Br}(\mu \to e\gamma). \tag{24}$$

For the μ-e conversion rate $\Gamma(\mu \to e)$ a similar relation holds at the $\tan\beta \gtrsim 1$ region,

$$\Gamma(\mu \to e) \simeq 16\alpha^4 Z_{\mathrm{eff}}^4 Z |F(q^2)|^2 \mathrm{Br}(\mu \to e\gamma). \tag{25}$$

Here Z is the proton number in the nucleus, and Z_{eff} is the effective charge, $F(q^2)$ the nuclear form factor at the momentum transfer q. The μ-e conversion rate normalized by the muon capture rate in Ti nucleus, $R(\mu^- \to e^-; {}^{48}_{22}\text{Ti})$, is approximately

$$R(\mu^- \to e^-; {}^{48}_{22}\text{Ti}) \simeq 6 \times 10^{-3} \text{Br}(\mu \to e\gamma). \tag{26}$$

The MECO collaboration proves that they have a technology to reach $R(\mu^- \to e^-; {}^{48}_{22}\text{Ti}) < 10^{-16}$[20]. Furthermore, now there are active discussions of the high intensity muon source, and we may reach to a level of 10^{-18} if the muon storage is constructed and $10^{(19-20)}$ muons per a year are produced[21]. This is comparable to $\text{Br}(\mu \to e\gamma) \sim 10^{-16}$, and we can probe the region $m_{\nu_\mu D} \simeq 0.2(0.5)\text{GeV}$ ($M_{\nu_\mu} \simeq 10^{10}$ $(10^{11})\text{GeV}$) in the MSW solution with the large angle.

5   Conclusion

In this article we discuss the charged lepton-flavor violating processes, $\mu^+ \to e^+\gamma$ and $\tau^\pm \to \mu^\pm\gamma$, using the current neutrino experimental data, in the SUSY SM with the right-handed neutrinos. While this model has many unknown parameters, these processes may be accessible in near future experiment. The LFV search will give new insights to the origin of the neutrino masses.

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